

**Cosmological Consequences of Conformal General Relativity**

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**Abstract**

We consider cosmological consequences of a conformal-invariant unified theory which is dynamically equivalent to general relativity and is given in a space with the geometry of similarity. We show that the conformal-invariant theory offers new explanations for to such problems as the arrow of time, initial cosmic data, dark matter and accelerating evolution of the universe in the dust stage.

**Introduction**

There are observations [1]-[6] that the classical equations of Einstein's general relativity (GR) are dynamically equivalent to the conformal-invariant theory described by the Penrose-Chernikov-Tagirov-type action [7] with a negative sign for the scalar (dilaton) field  $\Phi$  referred to as a conformal compensator. This dilaton version of GR (considered also as a particular case of the Jordan-Brans-Dicke scalar tensor theory of gravitation [8]) is the basis of some speculations on the unification of Einstein's gravity with the Standard Model of electroweak and strong interactions [1, 3, 6] including modern theories of supergravity [6]. In the conformal-invariant Lagrangian of matter, the dilaton generates the masses of elementary particles, i.e. it plays the role of the modulus of the Higgs field.

However, in the current literature [6] a peculiarity of the conformal-invariant version of Einstein's dynamics has been overlooked. The conformal-invariant version of Einstein's

dynamics is not compatible with the absolute standard of measurement of lengths and times given by an interval in the Riemannian geometry as this interval is not conformal-invariant. As it has been shown by Weyl in 1918 [9], conformal-invariant theories correspond to the relative standard of measurement of a conformal-invariant ratio of two intervals, given in the geometry of similarity as a manifold of Riemannian geometries connected by conformal transformations. The geometry of similarity is characterized by a measure of changing the length of a vector in its parallel transport. In the considered dilaton case, it is the gradient of the dilaton  $\Phi$  [3]. In the following, we call the scalar conformal-invariant theory the conformal general relativity (CGR) to distinguish it from the original Weyl [9] theory where the measure of changing the length of a vector in its parallel transport is a vector field (that leads to the defect of the physical ambiguity of the arrow of time pointed out by Einstein in his comment to Weyl's paper [9]). Thus, the choice between two dynamically equivalent theories — general relativity (GR) and conformal general relativity (CGR) is the choice between the Riemannian geometry and Weyl's geometry of similarity. Two different geometries for the same dynamics correspond to different standards of measurement and two different cosmological pictures for different observers: (I) an Einstein observer, who supposes that he measures an absolute interval of the Riemannian geometry, obtains the Friedmann-Robertson-Walker (FRW) cosmology where the redshift is treated as expansion of the universe; (II) a Weyl observer, who supposes that he measures a relative interval of the geometry of similarity, obtains a field version of the Hoyle-Narlikar cosmology [10]. The redshift and the Hubble law in the Hoyle-Narlikar cosmology [10] reflect the change of the size of atoms in the process of evolution of masses of elementary particles generated by the scalar dilaton field [2, 3, 10].

The present paper is devoted to a discussion of cosmological consequences of the conformal-invariant dilaton gravity.

## Conformal General Relativity

We start from the conformal-invariant theory described by the sum of the dilaton action and the matter action

$$W = W_{\text{CGR}} + W_{\text{matter}}. \quad (1)$$

The dilaton action is the Penrose-Chernikov-Tagirov one for a scalar (dilaton) field with the negative sign

$$W_{\text{CGR}}(g|\Phi) = \int d^4x \left[ -\sqrt{-g} \frac{\Phi^2}{6} R(g) + \Phi \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \Phi) \right]. \quad (2)$$

The conformal-invariant action of the matter fields can be chosen in the form

$$W_{\text{matter}} = \int d^4x \left[ \mathcal{L}_{(\Phi=0)} + \sqrt{-g}(-\Phi F + \Phi^2 B - \lambda \Phi^4) \right] , \quad (3)$$

where  $B$  and  $F$  are the mass contributions to the Lagrangians of the vector ( $v$ ) and spinor ( $\psi$ ) fields, respectively,

$$B = v_i(y_v)_{ij}v_j ; \quad F = \bar{\psi}_\alpha(y_s)_{\alpha\beta}\psi_\beta, \quad (4)$$

with  $(y_v)_{ij}$ , and  $(y_s)_{\alpha\beta}$  being the mass matrices of vector bosons and fermions coupled to the dilaton field. The massless part of the Lagrangian density of the considered vector and spinor fields is denoted by  $\mathcal{L}_{(\Phi=0)}$ . The class of theories of the type (1) includes the superconformal theories with supergravity [6] and the standard model with a massless Higgs field [3] as the mass term would violate the conformal symmetry.

Following Dirac we suppose that the symmetry of the theory establishes the symmetry of its observable quantities. In other words, the conformal invariance of the theory entails the conformal invariance of physical variables and measurable quantities.

Recall that in GR the problems of initial data and time evolution are studied with the help of the Lichnerowicz conformal-invariant variables  $F^L$  [11]

$$^{(n)}F^L = ^{(n)}F|{}^3g|^{-n/6}, \quad (5)$$

where  $^{(n)}F$  is a set of fields with a conformal weight  $(n)$  including the metric  $g_{\mu\nu}$  with  $|{}^3g^L_{ij}| = 1$ . The Lichnerowicz variables (5) are defined by the Dirac-ADM-foliation of the metric

$$(ds^L)^2 = g^L_{\mu\nu}dx^\mu dx^\nu = (N^L dt)^2 - {}^{(3)}g^L_{ij}\check{d}x^i\check{d}x^j, \quad \check{d}x^i = dx^i + N^i dt \quad (6)$$

with the lapse function  $N^L(t, \vec{x})$ , three shift vectors  $N^i(t, \vec{x})$ , and five space components  ${}^{(3)}g^L_{ij}(t, \vec{x})$  depending on the coordinate time  $t$  and the space coordinates  $\vec{x}$ . This definition excludes one superfluous degree of freedom and gives a conformal - invariant "measurable" interval in the CGR theory considered.

In terms of the Lichnerowicz variables (5), the dynamic equivalence of GR and CRG becomes evident

$$W_{\text{CGR}}[g^L|\Phi^L] = W_{\text{GR}}[g^L|\Phi_{\text{scale}}] . \quad (7)$$

Einstein's theory is obtained by replacing the Lichnerowicz dilaton field  $\phi^L$  with the new scale field

$$\Phi_{\text{scale}} \equiv M_{\text{Planck}} \sqrt{\frac{3}{8\pi}} |{}^3g|^{1/6}. \quad (8)$$

It can be possible by introducing the dimensional constant in the conformal -invariant theory (1). Therefore, the renunciation from the dimensional constant means the renunciation from the Einstein definition of measurable intervals in GR. Instead of the Einstein intervals we shall use the conformal - invariant Lichnerowicz intervals (6) without the determinant of the spatial metric (that disappears in the ratio of two intervals).

The opinion dominates that a dimensional constant (of the type of the Planck mass) can appear due to spontaneous conformal symmetry breaking in quantum perturbation theory; it can be a reason for introducing this constant in the theory from the very beginning [1, 6].

The formulation of the consistent reparametrization - invariant perturbation theory for Einstein's general relativity (GR) in Refs. [2, 3, 4, 5] gave a set of arguments in favor of the opposite point of view: the reparametrization - invariant perturbation theory does not violate the conformal symmetry. Therefore, in CGR, the role of the Planck mass is played by the dilaton field.

In the present paper we formulate conformal cosmology as a particular case of the conformal - invariant and reparametrization - invariant perturbation theory.

## Conformal - Invariant Theory of Cosmic Evolution

In the considered case of CGR (1) in terms of the conformal - invariant fields (5), perturbation theory begins from the homogeneous approximation for the dilaton and the metric

$$\Phi^L(t, x) = \varphi(t), \quad N^L(t, x) = N_0(t), \quad g_{ij}^L = \delta_{ij}, \quad (9)$$

which conserves the reparametrization - invariance even in the case of free conformal fields described by the action

$$W_0 = \int_{t_1}^{t_2} dt \left[ \varphi \frac{d}{dt} \frac{\dot{\varphi}}{N_0} V_0 + N_0 L_0 \right], \quad (10)$$

where  $V_0$  is a finite spatial volume.  $L_0$  is the sum of the Lagrangians of free fields,

$$L_0 = L_M + L_R + L_h, \quad (11)$$

where in particular

$$L_M = \frac{1}{2} \int_{V_0} d^3x \left( \frac{\dot{v}^2}{N_0^2} - (\partial_i v)^2 - (y_v \varphi)^2 v^2 \right) + \int_{V_0} d^3x \left( \bar{\psi} (-y_s \varphi - \frac{\gamma_0}{N_0} \partial_0 + i \gamma_j \partial_j) \psi \right) \quad (12)$$

is the Lagrangian of massive conformal fields (5) (massive vector ( $v$ ), and spinor ( $s$ )). The role of the masses is played by the homogeneous dilaton field  $\varphi$  multiplied by dimensionless

constants  $y_f$ , where  $f$  labels the particle species.  $L_R$  is the Lagrangian of massless fields (photons  $\gamma$ , neutrinos  $\nu$ ) with  $y_\gamma = y_\nu = 0$ , and

$$L_h = \int_{V_0} d^3x \frac{\varphi^2}{24} \left( \dot{h}^2 - (\partial_i h)^2 \right), \quad (13)$$

is the Lagrangian of the gravitons ( $h_{ii} = 0$ ;  $\partial_j h_{ji} = 0$ ) as weak transverse excitations of spatial metric (the last two equations follow from the unit determinant of the three-dimensional metric (5) and from the momentum constraint).

This reparametrization - invariant action (10) describes the well-known system of the free conformal fields in a finite space-volume used for studying the problem of creation of particles by the homogeneous excitation of the metric [4, 13, 14].

We propose that  $V_0$  coincides with the volume of the whole universe, so that a non-localizable energy [15] does not appear.

We call the system (10) with the invariant geometric time and the stationary metric

$$dT = N_0 dt, \quad ds_0^2 = dT^2 - dx_i^2, \quad (14)$$

a conformal - invariant universe. Our task is to find the evolution of all fields in the field world space  $(\varphi, f)$  with respect to the geometric time  $T$ .

The variation of the action with respect to the homogeneous lapse-function  $N_0$  yields the energy constraint,

$$\frac{\delta W_0}{\delta N_0} = 0 \Rightarrow \left( \frac{d\varphi}{N_0 dt} \right)^2 = \frac{H(\varphi)}{V_0} := \rho(\varphi), \quad (15)$$

where

$$H = \frac{\delta L_0}{\delta N_0} \quad (16)$$

is the Hamilton function of all field excitations with positive energy. Its expectation value determines the measurable energy density of all particles including gravitons (13), see below. The solution of equation (15),

$$T_\pm(\varphi_0) = \pm \int_0^{\varphi_0} d\varphi \rho^{-1/2}(\varphi), \quad (17)$$

describes the evolution of the dilaton (i.e. of all masses) with respect to the geometric time  $T$ .

Equation (15) as an energy constraint restricts the spectrum of values of the dilaton momenta to two solutions (with a positive sign and a negative one). In the equivalent unconstrained system (which can be constructed by substituting the solution of the Abelian

energy constraint (15) into the extended action [2, 4]) the dilaton momentum plays the role of the Hamiltonian of the evolution in  $\varphi$ .

Recall that in order to obtain a stable quantum theory, according to Dirac, one should propose that a universe with a positive energy (+) propagates forward with respect to the dynamic evolution parameter ( $\varphi_0 > \varphi_1$ ); and with a negative energy (-), backward ( $\varphi_0 < \varphi_1$ ). In both cases the geometric time (17) is always positive

$$T_+(\varphi_0 > \varphi_1) > 0, \quad T_-(\varphi_0 < \varphi_1) > 0. \quad (18)$$

The quantization of the dilaton field and the elimination of negative eigenvalues of the Hamiltonian (i.e. negative energies) by the Dirac treatment of the branch with the negative Hamiltonian as annihilation of universes with a positive energy immediately leads to the arrow of geometric time [5].

We introduce the particles as holomorphic field variables

$$f(t, \vec{x}) = \sum_{f,k} \frac{C_f(\varphi) \exp(ik_i x_i)}{V_0^{3/2} \sqrt{2\omega_f(\varphi, k)}} \left( a_f^+(-k, t) + a_f^-(k, t) \right), \quad (19)$$

where  $\omega_f(\varphi, k) = \sqrt{k^2 + y_f^2 \varphi^2}$  are the one-particle energies for the particle species  $f = h, \gamma, \nu, v, s$  with the dimensionless mass parameters  $y_f$  and the coefficients  $C_f(\varphi)$  are

$$C_h(\varphi) = \varphi \sqrt{12}, \quad C_\gamma = C_\nu = C_v = C_s = 1. \quad (20)$$

These variables diagonalize the operator of the observational density of matter in the well-known QFT form

$$\rho(\varphi) = \frac{H(\varphi)}{V_0} = \sum_{f,k} \frac{\omega_f(\varphi, k)}{V_0} N_f(\varphi, k), \quad (21)$$

where

$$\mathcal{N}_{a_f} = \langle |\{a_f^+, a_f\}_\pm| \rangle = \langle \left| \frac{1}{2}(a_f^+ a_f \pm a_f a_f^+) \right| \rangle \quad (22)$$

is the expectation value of the number of particles (the upper sign corresponds to bosons, the lower one to fermions),  $\langle |$  and  $| \rangle$  is the physical states determined by the initial cosmic data. In the following, we restrict ourselves to gravitons and massive vector particles.

This definition of particles (19) excludes the vertices of the dilaton - matter coupling which can restore the Higgs-type potential in the perturbation theory by the Coleman-Weinberg summing of the perturbation series. If we exclude from the very beginning the  $\lambda \phi^4$  term from the initial action in order to remove a tremendous vacuum density [16],

this term could not be restored by the perturbation series. Therefore, we suppose in the following that  $\lambda = 0$ .

Solutions of the equations of motion corresponding to the action of the system (10) - (13) can be obtained by a Bogoliubov transformation

$$b_f^+ = \cosh(r_f)e^{-i\theta_f}a_f^+ - i \sinh(r_f)e^{i\theta_f}a_f, \quad (23)$$

$$b_f = \cosh(r_f)e^{i\theta_f}a_f + i \sinh(r_f)e^{-i\theta_f}a_f^+, \quad (24)$$

where  $b_f^+$  and  $b_f$  are the creation and annihilation operators of Bogoliubov quasiparticles with  $N_f = \{b_f^+, b_f\}$  being the operator of the (conserved) numbers of quasiparticles [4].

We choose the initial state appropriate to the integrals of motion to be the quasiparticle vacuum state defined by  $b|0\rangle = 0$ . We call this state the "nothing" in order to distinguish it from the vacuum of observable particles. In this case, a set of equations for the expectation values of the numbers of particles (gravitons and mesons) are [4, 5, 17]

$$N_f = \frac{1}{2} \cosh(2r_f) = \langle 0 | \{a_f^+, a_f\} | 0 \rangle, \quad (25)$$

$$(\omega_f - \theta'_f) \sqrt{4N_f^2 - 1} = \Delta_f \cos(2\theta_f) 2N_f, \quad (26)$$

$$N'_f = -\Delta_f \sin(2\theta_f) \sqrt{4N_f^2 - 1}, \quad (27)$$

where the dash denotes the derivative  $d/dT$  with respect to the geometric time (14) and  $\Delta_h = \varphi'/\varphi$ ,  $\Delta_v = \omega'/(2\omega)$ .

The equations for the coefficients of the Bogoliubov transformations can be solved explicitly in two limits: at the beginning of the Universe and at the present-day stage [4].

At the beginning of the Universe in the state of the Bogoliubov (i.e. squeezed) vacuum of quasiparticles, we got the density of measurable gravitons (particles) which corresponds to the well-known anisotropic (Kasner) stage with the Misner wave function of the Universe [18]. The anisotropic stage is followed by the stage of inflation-like increase in the cosmic scale factor with respect to the time measured by an observer with the relative standard. At these stages, the Bogoliubov quasiparticles strongly differ from the measurable particles.

## Observational consequences of Conformal Cosmology

The initial action (10) shows that all masses of particles increase with the geometric time. A photon emitted by an atom of a star two billion years ago remembers the size of that atom (i.e., its mass) at the time of emission ( $T_0 - D$ ). After two billion years, at the time

of detection on the earth ( $T_0$ ), the wavelength of a photon is compared to the wavelength of a photon emitted by a standard atom on the Earth when its size decreased due to the cosmic evolution of the masses of elementary particles. This is just the version of cosmology proposed by Hoyle and Narlikar [10] where the origin of the redshift

$$Z(D) = \frac{\varphi(T_0)}{\varphi(T_0 - D/c)} - 1 = \mathcal{H}_0(T_0)D/c + \dots, \quad (28)$$

is the evolution of particle masses determined in our case by the dilaton  $\varphi$ . The next step is the identification of the conformal quantities of the Hoyle-Narlikar cosmology (geometric time  $T$ , distance  $D$ , density of the matter-energy  $\rho(\varphi) = H_0/V_0$ , Hubble's parameter  $\mathcal{H}_0(T_0) = \varphi'(T_0)/\varphi(T_0)$ ) with the observational ones.

The evolution of the dilaton is determined by equation (15)

$$\varphi' \equiv \frac{d\varphi}{dT} = \sqrt{\rho(\varphi)}, \quad (29)$$

where  $\rho(\varphi)$  is considered as a measurable density of matter in the conformal cosmology. We can express the present-day value of the dilaton  $\varphi(T_0)$  in terms of observational quantities: the density  $\rho$  and the Hubble parameter

$$\varphi(T_0) = \frac{\sqrt{\rho(T_0)}}{\mathcal{H}_0}. \quad (30)$$

The cosmological observational data for the density parameter  $\Omega = \rho/\rho_c$  with  $\rho_c = 3\mathcal{H}_0^2 M_{\text{Planck}}^2/(8\pi)$  allow us to assert that the present value of the dilaton field is related to the Planck mass,

$$\varphi(T_0) = M_{\text{Planck}} \sqrt{\frac{3}{8\pi}}, \quad (31)$$

for  $\Omega \approx 1$ , see below. Thus, on the fundamental level of CGR, the Planck mass is not a fundamental constant, but determined by the value of the dilaton field  $\varphi(T)$  (31). This is a difference of principle between CGR and Einstein's GR. One can relate both theories by fixing an interval of the absolute (conformal - noninvariant) world time in Einstein's theory

$$dT_f = a(T)dT \quad \left( a(T) = \frac{\varphi(T)}{M_{\text{Planck}}} \sqrt{\frac{8\pi}{3}} \equiv \frac{\varphi(T)}{\varphi(T_0)} \right). \quad (32)$$

Einstein's theory supposes that an observer measures this absolute interval in Riemannian space. As a result, he obtains the Friedmann-Robertson-Walker (FRW) cosmology where the redshift is treated as expansion of the universe and the measurable density  $\rho^{\text{exp}}$  is identified with the Einstein density  $\rho_{\text{Einstein}} = \rho^{\text{exp}}/a^4$ . In the FRW cosmology the



experimental fit (30) is treated as a standard definition of the critical density provided  $a(T_0) = 1$ . The coincidence of the values of the scale factors in both the versions of cosmology does not mean the equivalence of their dynamic evolution in corresponding times. In the FRW version, the mass density is decreasing, whereas in the conformal version, the mass density is increasing.

Thus, in CGR, a large wave-length of a photon  $\lambda_S = [y\varphi(T_0 - D/c)]^{-1}$  emitted from an atom in a star at distance  $D$  is compared to a small one  $\lambda_E = [y\varphi(T_0)]^{-1}$  corresponding to an atom on earth within a stationary universe. In GR, the wave-length of a photon from a distant star  $\lambda_S$  becomes greater by a factor of  $a(T_0)/a(T_0 - D/c)$  due to the cosmic evolution of all length during its travel to the observer on Earth. In both the cases we obtain a redshift  $Z > 1$ . In CGR,  $Z > 0$  is explained by the increasing atomic masses. In GR,  $Z > 0$  is explained by increasing the star photon wave-length during its flight.

To discuss the problem of dark matter in the conformal cosmology, we should also take into account that the present-day observations reflect the matter density  $\Omega(T_0 - D/c)$  at the time when the light was emitted from the cosmic objects. The mass density was less than at the present-day value  $\Omega(T_0) = \Omega_0$  due to the mass increase of the particles by the dilaton field  $\varphi(T)$  with progressing conformal time. This effect of the retardation in the matter density can be roughly estimated by averaging  $\Omega(T_0 - D/c)$  over distances (or time) introducing the coefficient

$$\gamma = \frac{T_0 \Omega_0}{\int_0^{T_0} dT \Omega(T)}.$$

For the dust stage the coefficient of the mass increase is  $\gamma = 3$ . We get for the present-day value of the cosmic matter density in CGR and the Planck "constant" in GR

$$\Omega(T_0) = \gamma \Omega_0^{\text{exp}} \approx 1, \quad (33)$$

where the value of  $\Omega_0^{\text{exp}} \approx 0.3$  has been taken from a recent analysis of the total luminous and dark matter density [19]. This result would entail that in a flat universe case, there is no reason for dark energy, since the missing energy density problem occurs only due to the neglect of this mass retardation effect in the standard analysis of cosmological parameters. The final question arises: Can we account within the conformal cosmology scenario for the observed cosmic acceleration without the cosmological constant or quintessence models?

At the present-day stage, the Bogoliubov quasiparticles coincide with the measurable particles, so that the measurable density of energy of matter in the universe is a sum of

relativistic energies of all particles in it (with the number of particles  $N_{n,f}(k)$ )

$$\rho(\varphi) = \sum_{f,k_n} \sqrt{k_n^2 + y_f^2 \varphi^2} \frac{N_{n,f}(k_n)}{V_0} \quad (34)$$

The wave-function of the Universe  $\Psi_{\text{univ.}}$  is nothing but the product of oscillator wave-functions  $\Psi_{\text{univ.}} = \prod \Psi_{\text{part.}} = \prod \exp(iT \sqrt{k_n^2 + y_f^2 \varphi^2})$ .

Neglecting masses ( $y_f = 0$ ), we get the conformal version of the radiation stage ( $\rho(\varphi) = \bar{\rho}_R$ ) for an observer with the relative standard. The evolution law for a scalar field in this case is

$$\varphi(T_0) = \sqrt{\bar{\rho}_R} T_0, \quad (35)$$

and the Hubble parameter  $\mathcal{H}_0(T_0)$  is

$$\mathcal{H}_0 = \frac{1}{\varphi} \frac{d\varphi}{dT_0} = \frac{1}{T_0}. \quad (36)$$

Neglecting momenta ( $k_n = 0$ ), we get the conformal version of the dust stage ( $\rho(\varphi) = \varphi \bar{\rho}_D$ ) with the evolution law

$$\varphi(T_0) = \frac{T_0^2}{4} \bar{\rho}_D, \quad (37)$$

the Hubble parameter  $\mathcal{H}_0(T_0)$

$$\mathcal{H}_0 = \frac{\varphi'}{\varphi} = \frac{2}{T_0}, \quad (38)$$

and the acceleration-parameter

$$q = -\frac{\varphi'' \varphi}{\varphi'^2} = -\frac{1}{2}, \quad (39)$$

which is in agreement with the recent data from the Supernova Cosmology Project [20].

## Summary

In this paper, we have emphasized the conformal-invariant treatment of the GR dynamics is compatible with the Weyl geometry of similarity but not with the Riemannian one. The geometry of similarity converts the conformal-invariant Lichnerowicz variables from an effective mathematical tool to physical observables.

The important consequence of the geometry of similarity is the conformal cosmology.

The present-day value of the dilaton expressed in terms of observational data of the field version of the Hoyle-Narlikar cosmology coincides with the Planck mass within the limits of observational errors.

The conformal cosmology and its quantum version allow us to give answers to a set of problems of the standard cosmology including the positive arrow of the geometric time (as a consequence of the positive energy of a dynamic system and of its stability), the anisotropic inflation stage, the creation of a universe as a dynamical system in the field world space, and as well as the retardation origin of dark matter (as we estimate the present-day mass density using data of the earlier stages where the mass density was smaller). We would like to emphasize that the phenomenon of the accelerating evolution of the universe in the dust stage is in agreement with the present observational data.

## Acknowledgments

The authors are grateful to B. Barbashov, and M. Pawlowski for fruitful discussions. D.B. and D.B. acknowledge support from the Heisenberg-Landau program for visiting the JINR Dubna where part of this work has been done.

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